frequency was obtained in the final module.[1] This value reflects some degradation of BARITT self-mixing sensitivity from −132 dB/Hz achieved in the circuit shown in Fig. 6. This was partly due to nonoptimized device-circuit condition. The circuit had to be adjusted to match the resonant frequency of the narrow-band microstrip antenna.

V. Conclusion

An X-band MIC BARITT self-mixing oscillator has been developed with detection sensitivity comparable to that obtained with the coaxial cavity. A minimum detectable signal of −139 dB/Hz below carrier was achieved at 100 kHz away from the carrier. A compact, low cost and sensitive hybrid MIC Doppler sensor module has been constructed incorporating the BARITT MIC circuit and a microstrip antenna.

[1]The MDSBC was obtained from the measured range capability of the Doppler radar module.

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On the Use of a Microstrip Three-Line System as a Six-Port Reflectometer

RICHARD J. COLLIER AND NABIL A. EL-DEEB

***Abstract*—The scattering parameters for a coupled symmetrical three-line system in an inhomogeneous dielectric medium (e.g., microstrip) are derived directly in terms of a set of three orthogonal modes. The obtained results show that the condition for isolation of nonadjacent ports (e.g., ports 1 and 3 in Fig. 1) does not result from putting the corresponding per unit length immittance parameters equal to zero (i.e., $z_{13}=y_{13}=0$). The use of such a three-line system as a six-port reflectometer is analyzed in terms of the derived scattering parameters. The reflectometer discussed in this paper allows an unknown impedance to be measured using a standard impedance.**

I. Introduction

The properties of coupled multiconductor systems have been extensively investigated both for homogeneous [1]–[3] and inhomogeneous [4]–[7] media. Most of the introduced analyses were based on the use of either the capacitance or the immittance matrix of the system. In many applications, e.g., analysis of couplers and reflectometers, the use of the scattering parameters of the system gives a more physical insight into the problem. The scattering parameters were used only partly for the analysis of a three-line coupler [5]. In this paper a more detailed analysis of a symmetrical three-line system (Fig. 1), which was analyzed in terms of the per unit length immittances [6], is presented in terms of the system's scattering parameters. These scattering parameters are derived in Section II. In Section III it is shown that the condition for isolation of nonadjacent ports, e.g., ports 1 and 3 in Fig. 1, is not met by putting the corresponding per unit length immittances equal to zero, i.e., $z_{13}=y_{13}=0$. In Section IV the necessary conditions allowing the use of the three-line system as one class of six-port reflectometers are derived making use of the results of the previous sections. In Section V an investigation is carried out to find the most

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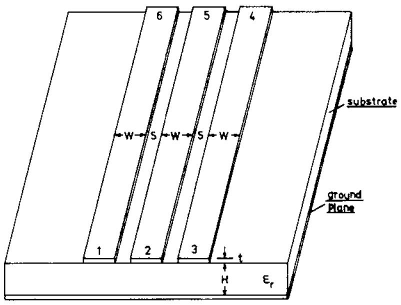

Fig. 1. A microstrip symmetrical three-line system.

suitable configuration for such a reflectometer, and a proposed configuration is given.

II. Derivation of the Scattering Parameters

TEM propagation along a coupled set of N parallel and infinitely long lines having uniform cross sections, and above a common-ground plane, can be described in terms of N orthogonal modes [8]. The orthogonal modes that will be used are similar to those of [6] and are based on the solution of telegraphist's equations of the system. From this solution the following voltage and current eigenvectors matrices, for the case of three lines, can be obtained:

$$[M_V]=\begin{bmatrix} 1 & 1 & 1 \\ 0 & m_1 & m_2 \\ -1 & 1 & 1 \end{bmatrix} \tag{1a}$$

$$[M_I]=\begin{bmatrix} 1 & 1 & 1 \\ 0 & n_1 & n_2 \\ -1 & 1 & 1 \end{bmatrix}. \tag{1b}$$

The possible voltage modes of propagation according to (1a) are given by

$$M_A=\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \qquad M_B=\begin{bmatrix} 1 \\ m_1 \\ 1 \end{bmatrix} \qquad M_C=\begin{bmatrix} 1 \\ m_2 \\ 1 \end{bmatrix} \tag{2}$$

and will be called, respectively, mode A, mode B, and mode C.

The scattering parameters will be derived by using a system of voltage generators [9], [10] which allows a separate excitation for each of the propagation modes. This enables each line to be treated, for each of these modes, as a two-port network. The coefficients of reflection Γ and transmission T for each line, in a lossless and quasi-homogeneous medium, are given by [5], [10]

$$\Gamma_x=j\left[\left[\frac{Z_x}{Z_0}-\frac{Z_0}{Z_x}\right]\sin\theta_x\right]\Big/\phi_x \tag{3}$$

and

$$\Gamma_x=2/\phi_x \tag{4}$$

where $\phi_x=2\cos\theta_x+j[(Z_x/Z_0)+(Z_0/Z_x)]\sin\theta_x$, x stands for the mode under consideration (mode A, B, or C), θ_x is the electrical length of the line for this mode, and Z_0 is the system's characteristic impedance.

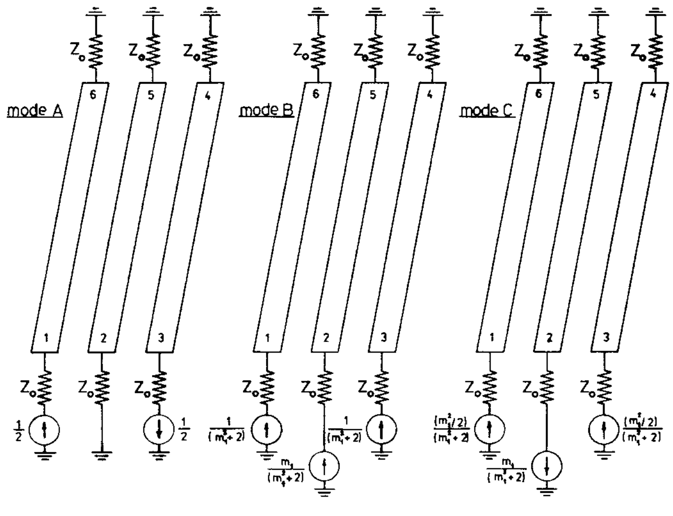

Fig. 2. The system of generators used to derive the s-parameters for an overall input signal of 1 V at port 1.

For each of the propagation modes, the mode characteristic impedances of the three lines are set equal [5], [6]. The characteristic impedances for modes A, B, and C will be called Z_A, Z_B, and Z_C, respectively.

The scattering parameters relating the emerging voltage waves at all ports (assumed matched) to an incident voltage wave at port 1 can be determined by using the system of voltage generators, shown in Fig. 2. Their voltages should satisfy (2) and when superimposed at each port result in an overall input voltage of 1 V at port 1 and zero at ports 2 and 3.

Since for each mode the characteristic impedances for the three lines were considered equal, the reflection and transmission coefficients for each mode are the same on each of the three lines. Thus the scattering parameters for this case are given by

$$\begin{aligned}
S_{11}&=\tfrac{1}{2}\Gamma_A+\frac{1}{(m_1^2+2)}\Gamma_B+\frac{m_1^2}{2(m_1^2+2)}\Gamma_C\\
S_{21}&=\frac{m_1}{(m_1^2+2)}[\Gamma_B-\Gamma_C]\\
S_{31}&=-\tfrac{1}{2}\Gamma_A+\frac{1}{(m_1^2+2)}\Gamma_B+\frac{m_1^2}{2(m_1^2+2)}\Gamma_C\\
S_{41}&=-\tfrac{1}{2}T_A+\frac{1}{(m_1^2+2)}T_B+\frac{m_1^2}{2(m_1^2+2)}T_C\\
S_{51}&=\frac{m_1}{(m_1^2+2)}[T_B-T_C]\\
S_{61}&=\tfrac{1}{2}T_A+\frac{1}{(m_1^2+2)}T_B+\frac{m_1^2}{2(m_1^2+2)}T_C\,.
\end{aligned} \tag{5}$$

Because of the symmetry of ports 1 and 3 about port 2, the scattering parameters relating the emerging voltage waves at all ports to an incident voltage wave at port 3 will be similar to those of port 1, i.e., $S_{13}=S_{31}$, $S_{23}=S_{21}$,

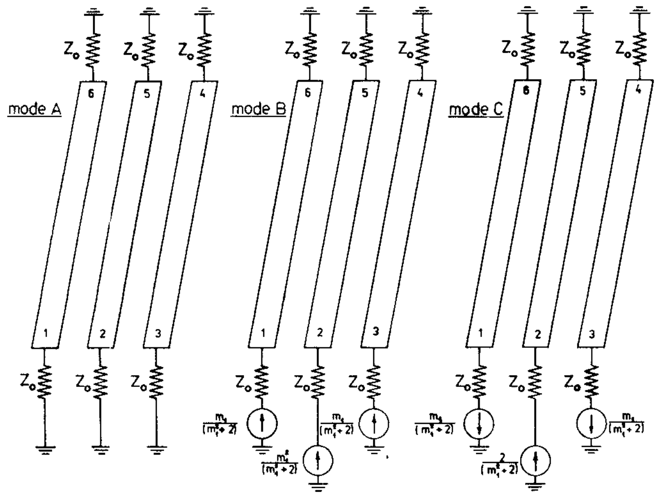

Fig. 3. The system of generators used to derive the s-parameters for an overall input signal of 1 V at port 2.

$S_{33}=S_{11}$, $S_{43}=S_{61}$, $S_{53}=S_{51}$, and $S_{63}=S_{41}$. They can be derived using the system of generators of Fig. 2 with the only exception of exchanging the generators connected to ports 1 and 3 for mode-A excitation.

The scattering parameters relating the emerging voltage waves at all ports to an incident voltage wave at port 2 can be determined using the system of generators, shown in Fig. 3. Their voltages satisfy the previously mentioned conditions, no mode-A excitation is required for this case, and the scattering parameters are given by

$$S_{12}=\frac{m_1}{(m_1^2+2)}[\Gamma_B-\Gamma_C]$$
$$S_{22}=\frac{m_1^2}{(m_1^2+2)}\Gamma_B+\frac{2}{(m_1^2+2)}\Gamma_C$$
$$S_{32}=\frac{m_1}{(m_1^2+2)}[\Gamma_B-\Gamma_C]$$
$$S_{42}=\frac{m_1}{(m_1^2+2)}[T_B-T_C]$$
$$S_{52}=\frac{m_1^2}{(m_1^2+2)}T_B+\frac{2}{(m_1^2+2)}T_C$$
$$S_{62}=\frac{m_1}{(m_1^2+2)}[T_B-T_C]. \tag{6}$$

Because of the symmetry of the system the relations at ports 4, 5, and 6 are the same as the corresponding ones at ports 1, 2, and 3.

The whole scattering parameters can be summarized in the following:

$$S_{11}=S_{33}=S_{44}=S_{66}=\alpha$$
$$S_{12}=S_{21}=S_{23}=S_{32}=S_{45}=S_{54}=S_{56}=S_{65}=\beta$$
$$S_{13}=S_{31}=S_{46}=S_{64}=\gamma$$
$$S_{14}=S_{41}=S_{36}=S_{63}=\delta$$

and

$$S_{15}=S_{51}=S_{35}=S_{53}=S_{24}=S_{42}=S_{26}=S_{62}=\epsilon$$
$$S_{16}=S_{61}=S_{34}=S_{43}=J$$
$$S_{22}=S_{55}=\xi$$
$$S_{25}=S_{52}=\tau. \tag{7}$$

The scattering matrix of the system will thus have the form

$$[S]=\left[\begin{array}{ccc|ccc}\alpha & \beta & \gamma & \delta & \epsilon & J\\ \beta & \xi & \beta & \epsilon & \tau & \epsilon\\ \gamma & \beta & \alpha & J & \epsilon & \delta\\ \hline \delta & \epsilon & J & \alpha & \beta & \gamma\\ \epsilon & \tau & \epsilon & \beta & \xi & \beta\\ J & \epsilon & \delta & \gamma & \beta & \alpha\end{array}\right]. \tag{8}$$

III. Conditions for Isolation of Nonadjacent Ports

In much of the literature dealing with a system of coplanar parallel lines either capacitances or immittances (per unit length) between nonadjacent lines are considered relatively small, and hence neglected. However, for some applications, such as the three-line coupler of [5] and the reflectometer, to be discussed in the next section, it is necessary to know either the amount of coupling between these ports or the condition leading to their isolation. This section shows that the condition of isolation between ports 1 and 3 (or 4 and 6) in Fig. 1, based on the results of Section II, is not met by merely setting the corresponding (per unit length) immittances to zero (i.e., $z_{13}=y_{13}=0$).

From (5) and (7) it follows that for isolation of ports 1 and 3 (or 4 and 6), (Fig. 1), i.e., $S_{13}=S_{31}=S_{46}=S_{64}=0$, the following condition should be satisfied:

$$\Gamma_A=\frac{2}{(m_1^2+2)}\Gamma_B+\frac{m_1^2}{(m_1^2+2)}\Gamma_C \tag{9}$$

when the expressions for Γ_A, Γ_B, and Γ_C, for a coupling region of $\lambda/4$ in a lossless quasi-homogeneous medium, are substituted in (9) we get

$$Z_0=\sqrt{\frac{Z_B^2\{m_1^2Z_A^2-(m_1^2+2)Z_C^2\}+2Z_A^2Z_C^2}{2Z_B^2-(m_1^2+2)Z_A^2+m_1^2Z_C^2}}. \tag{10}$$

Expression (10) reduces to that of [5] (Section IV, (21)) for $\Gamma_A=0$ (or identically $Z_A=Z_0$).

The mode impedances can be derived as given in [6], but with $y_{22}=y_{11}+y_{13}$, and $z_{22}=z_{11}+z_{13}$ (i.e., $z_{13}\neq y_{13}\neq 0$), they are as follows:

$$Z_A=\sqrt{\frac{z_{11}-z_{13}}{y_{11}-y_{13}}} \tag{11}$$

$$Z_B=\sqrt{\frac{z_{11}+z_{13}+\sqrt{2}\,z_{12}}{y_{11}+y_{13}+\sqrt{2}\,y_{12}}} \tag{12}$$

$$Z_C=\sqrt{\frac{z_{11}+z_{13}-\sqrt{2}\,z_{12}}{y_{11}+y_{13}-\sqrt{2}\,y_{12}}}\ . \tag{13}$$

The conditions upon which the mode impedances above were derived [6] result in a value of $\sqrt{2}$ for m_1. When this value of m_1 is substituted in (10) together with the expressions for Z_A, Z_B, and Z_C from (11), (12), and (13), we get

$$Z_0=\sqrt{\frac{z_{12}(y_{11}z_{12}-z_{11}y_{12})-z_{13}(z_{13}y_{11}-z_{12}y_{12})+y_{13}(z_{11}^2-z_{12}^2)+y_{13}z_{13}z_{11}}{y_{12}(z_{11}y_{12}-y_{11}z_{12})-y_{13}(y_{13}z_{11}-y_{12}z_{12})+z_{13}(y_{11}^2-y_{12}^2)+z_{13}y_{13}y_{11}}}\ . \tag{14}$$

Thus (14) represents the condition for isolation of the nonadjacent ports 1 and 3.

If z_{13} and y_{13} are put equal to zero now, condition (14) becomes

$$Z_0=\sqrt{\frac{z_{12}}{-y_{12}}}\ . \tag{15}$$

This shows that in case of $z_{13}=y_{13}=0$, condition (15) should still be satisfied to have isolation of the considered ports.

The element y_{12} in (15), and so all the off-diagonal elements of the admittance matrix in [6] should have negative numerical values since they represent the inter-lines admittances for a system of lines above a common ground plane. This latter fact is also illustrated in [4].

IV. The Use of the Three-Line System as a Six-Port Reflectometer

If the three-line system fulfills certain requirements, it can be used as one class of six-port reflectometers [11]. Such a reflectometer allows an unknown impedance to be measured by using a standard impedance. An ideal performance of such a reflectometer could be attained if all ports are matched and nonadjacent ports, ports 1–3 and 4–6 in Fig. 1, are isolated. However, for the considered three-line system, the above conditions cannot be met all at the same time (see Appendix). Thus we are left with the following three possible configurations.

1) Matching at all ports, and in this case we will have transmission from each port to all other ports.
2) Matching at side ports, ports 1, 3, 4, and 6 (Fig. 1), and no transmission between ports 1⇄5, 2⇄4, 2⇄6, 3⇄5, i.e., $S_{15}=S_{51}=S_{24}=S_{42}=S_{26}=S_{62}=S_{35}=S_{53}=0$.
3) Matching at side ports, ports 1, 3, 4, and 5 (Fig. 1), and isolation between nonadjacent ports, ports 1⇄3 and 4⇄6, i.e., $S_{13}=S_{31}=S_{46}=S_{64}=0$.

In the considered reflectometer the unknown and the standard impedances should be connected to two ports isolated from each other, to avoid interreflections between them that would disturb the reflectometer performance. Thus the first configuration is clearly unsuitable for the intended performance of the reflectometer.

The second configuration is suitable for our application, and here it is required that $S_{11}=S_{33}=S_{44}=S_{66}=0$, and $S_{15}=S_{51}=S_{35}=S_{53}=S_{24}=S_{42}=S_{26}=S_{62}=0$. The latter requirement can be obtained by putting $T_B=T_C$ in (5) and (6).

For $T_B=T_C$, (3) and (4) give $\Gamma_B=-\Gamma_C$, and when this is substituted in the expression for S_{11} in (5), and the latter is set to zero, we get

$$\Gamma_A=\frac{(m_1^2-2)}{(m_1^2+2)}\Gamma_B. \tag{16}$$

For a lossless and quasi-homogeneous medium the condition $\Gamma_B=-\Gamma_C$ leads to $Z_BZ_C=Z_0^2$, and from (16) m_1 can be expressed in terms of the mode impedances Z_A and Z_B as

$$m_1=\frac{\sqrt{2}}{Z_0}\sqrt{\frac{(Z_A^2Z_B^2-Z_0^4)}{(Z_B^2-Z_A^2)}}\ . \tag{17}$$

The scattering parameters of the system (at the center frequency) are given by

$$\begin{aligned}
&S_{11}=S_{33}=S_{44}=S_{66}=0\\
&S_{12}=S_{21}=S_{23}=S_{32}=S_{45}=S_{54}=S_{56}=S_{65}\\
&\quad=\frac{2m_1}{(m_1^2+2)}\Gamma_B=\beta\\
&S_{13}=S_{31}=S_{46}=S_{64}=\frac{(2-m_1^2)}{(2+m_1^2)}\Gamma_B=-\Gamma_A=\gamma\\
&S_{14}=S_{41}=S_{36}=S_{63}=\tfrac{1}{2}(T_B-T_A)\\
&\quad=-\frac{1}{2}j\left[\frac{2Z_0Z_B}{(Z_0^2+Z_B^2)}-\frac{2Z_0Z_A}{Z_0^2+Z_A^2}\right]=\delta\\
&S_{15}=S_{51}=S_{35}=S_{53}=S_{24}=S_{42}=S_{26}=S_{62}=0\\
&S_{16}=S_{61}=S_{34}=S_{43}=-\frac{1}{2}j\left[\frac{2Z_0Z_B}{(Z_0^2+Z_B^2)}+\frac{2Z_0Z_A}{(Z_0^2+Z_A^2)}\right]=J\\
&S_{22}=S_{55}=\frac{(m_1^2-2)}{(m_1^2+2)}\Gamma_B=\Gamma_A=\xi\\
&S_{25}=S_{52}=T_B=-j\frac{2Z_0Z_B}{(Z_0^2+Z_B^2)}
\end{aligned} \tag{18}$$

where

$$\Gamma_B=\frac{Z_B^2-Z_0^2}{Z_B^2+Z_0^2}. \tag{19}$$

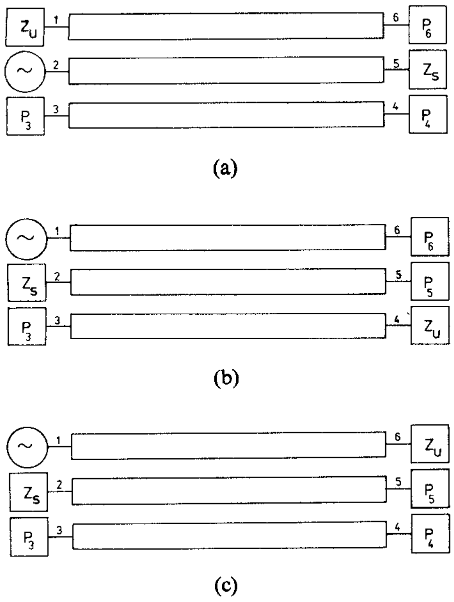

Fig. 4. The three investigated connections of the second possible configuration.

In the present case it is required that the unknown and the standard impedances are connected to two isolated ports with the unknown connected to one of the matched ports (port 1, 3, 4, or 6). Three possible connections satisfying these requirements are shown in Fig. 4.

The principle of operation of the three-line system as a six-port reflectometer will be illustrated in conjunction with the connection given in Fig. 4(a). For the connections in Fig. 4(b) and (c) only final results will be given which can be easily deduced following the same procedure.

Throughout the following analyses it is assumed that the used power meters and generator are matched to the respective ports to which they are connected.

In Fig. 4(a) a generator is connected to port 2, an unknown Z_u (of reflection coefficient Γ_u) to port 1, a standard impedance Z_s (of reflection coefficient Γ_s) to port 5, and three power meters P_3, P_4, and P_6 to ports 3, 4, and 6, respectively.

From (8) and (18) the emerging voltage waves at ports 3, 4, and 6 can be determined and are given, respectively, by

$$\begin{aligned} b_3 &= \beta\gamma\Gamma_u a_0 + \beta a_0 \\ b_4 &= \beta\delta\Gamma_u a_0 + \beta\tau\Gamma_s a_0 \\ b_6 &= \beta J\Gamma_u a_0 + \beta\tau\Gamma_s a_0 \end{aligned} \tag{20}$$

where a_0 is the incident voltage wave at port 2.

The output powers P_3, P_4, and P_6 are thus given by

$$\begin{aligned} P_3 &= |b_3|^2 = |\beta\gamma|^2 P_0 \left|\Gamma_u + \frac{1}{\gamma}\right|^2 \\ P_4 &= |b_4|^2 = |\beta\delta|^2 P_0 \left|\Gamma_u + \Gamma_s\frac{\tau}{\delta}\right|^2 \\ P_6 &= |b_6|^2 = |\beta J|^2 P_0 \left|\Gamma_u + \Gamma_s\frac{\tau}{J}\right|^2 \end{aligned} \tag{21}$$

where $P_0 = |a_0|^2$ is the input power at port 2. If P_0, P_3, P_4, and P_6 are constant, (21) represent three circles in the Γ_u plane having centers at $-1/\gamma$, $-\Gamma_s(\tau/\delta)$, and $-\Gamma_s(\tau/J)$. These three circles intersect at a point which determines Γ_u both in magnitude and phase.

For the connection of Fig. 4(b), we have

$$\begin{aligned} P_3 &= |\delta J|^2 P_0 \left|\Gamma_u + \left(\Gamma_s\frac{\beta^2}{\delta J} + \frac{\gamma}{\delta J}\right)\right|^2 \\ P_5 &= |\delta\beta|^2 P_0 \left|\Gamma_u + \Gamma_s\frac{\tau}{\delta}\right|^2 \\ P_6 &= |\delta\gamma|^2 P_0 \left|\Gamma_u + \frac{J}{\gamma\delta}\right|^2. \end{aligned} \tag{22}$$

The centers of the circles given by (22) for constant P_3, P_5, P_6, and P_0 are

$$-\left(\Gamma_s\frac{\beta^2}{\delta J} + \frac{\gamma}{\delta J}\right), \quad -\Gamma_s\frac{\tau}{\delta}, \quad \text{and} \quad -\frac{J}{\gamma\delta}.$$

For the connection of Fig. 4(c), we have

$$\begin{aligned} P_3 &= |\delta J|^2 P_0 \left|\Gamma_u + \left(\frac{\gamma}{\delta J} + \Gamma_s\frac{\beta^2}{\delta J}\right)\right|^2 \\ P_4 &= |\gamma J|^2 P_0 \left|\Gamma_u + \frac{\delta}{\gamma J}\right|^2 \\ P_5 &= |\beta J|^2 P_0 \left|\Gamma_u + \Gamma_s\frac{\tau}{J}\right|^2. \end{aligned} \tag{23}$$

The centers of the circles given by (23) for constant P_3, P_4, P_5, and P_0 are

$$-\left(\frac{\gamma}{\delta J} + \Gamma_s\frac{\beta^2}{\delta J}\right), \quad -\frac{\delta}{\gamma J}, \quad \text{and} \quad -\Gamma_s\frac{\tau}{J}.$$

The third configuration has been already used to design a six-port directional coupler [5]. For this case two connections are possible with a generator connected to port 2 and the unknown and standard impedances connected either to ports 1 and 3 or ports 4 and 6. When the centers of the circles corresponding to these connections were determined using the previously outlined procedure, they were not found to be suitably positioned for the reasons that will be given briefly in the next section.

V. Determination of the Most Suitable Connection

Since the unknown reflection coefficient Γ_u is determined by the intersection of three circles, the accuracy of its determination will depend largely on how the centers of these circles are situated relative to the origin and relative to each other. By the most suitable connection we mean the one giving the best compromise between the above requirements as compared to the other possible connections.

Moderate values of coupling ($\simeq$10 dB) between adjacent ports (e.g., ports 1 and 2 in Fig. 1) can be easily

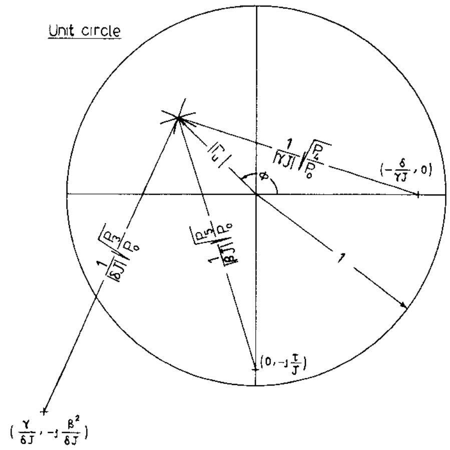

Fig. 5. Determination of the unknown reflection coefficient (impedance) from the readings of the three power meters of Fig. 4(c).

realized in a microstrip form using alumina substrates. For this coupling level the scattering parameters of the system can be determined using the previously derived expressions and the design information of [5]. From these determined parameters it can be seen easily that the quantities $1/\gamma$ and τ/δ are relatively large compared with unity. Thus the centers of the first and second circles, (21), related to the connection of Fig. 4(a) will lie far from the origin of the Γ_u plane. The center of the third circle $-\Gamma_s(\tau/J)$ lies near the boundary of the unit circle, and it is the most reasonably situated one out of the three centers. Similarly, the centers $-\Gamma_s(\tau/\delta)$ and $-J/\gamma\delta$ for the second and third circles, (22), related to the connection of Fig. 4(b) will lie far from the origin of the Γ_u plane, while that of the first circle lies reasonably far from the origin. Thus these two connections (Fig. 4(a) and (b)), are not suitable for our application. For the same reasons the two connections of the third configuration, that have been mentioned at the end of the previous section, are not suitable as well. This leaves us with the connection of Fig. 4(c) and it is obvious from its related equation (23) that the expressions for the centers of the three circles provide the most reasonable positions compared with the other connections.

If the standard impedance connected to port 2, Fig. 4(c) is chosen as a standard short circuit offset by $\lambda/8$ (at the center frequency) from the reference plane at that port, then the center of the circle whose radius is proportional to the output power P_3, (23), is given by $(-(\gamma/\delta J)-j(\beta^2/\delta J))$. The center of the circle whose radius is proportional to the output power P_4 (23) is given by $-\delta/\gamma J$. While the center of the circle whose radius is proportional to the output power P_5 (23) is given by $-j(\tau/J)$. The last two centers are expected to be near to the boundary of the unit circle, while the first center is expected to lie outside the unit circle and reasonably far from the origin of the Γ_u plane. The situation is shown in Fig. 5 together with the way of the determining of the reflection coefficient Γ_u, both in magnitude and phase, of the unknown impedance connected to port 6, Fig. 4(c). If a Smith chart of the suitable scale is overlaid on the unit circle, then Z_u can be directly determined.

VI. Conclusions

The scattering parameters for a three-line system in an inhomogeneous medium have been derived. By using these parameters, it has been shown that the condition of no coupling between nonadjacent ports is not met by only setting the corresponding per unit length immittances to zero. Making use of the derived scattering parameters, a theory was developed for the application of the three-line system as a reflectometer. Investigations for the most suitable configuration for such a reflectometer have been presented and lead to the determination of a proposed configuration. Then the expected positions for the centers of the three circles, using this configuration, have been illustrated together with the way of determining the reflection coefficient of the unknown impedance or the impedance itself. The presented analyses and theory show clearly the advantages of using the scattering parameters concept in such types of problems. The results of the experimental work based on the theory given in this paper will be presented in a following paper.

Appendix

I. *Expressing the mode impedances in terms of the system's capacitances*:

The capacitance matrix of the considered symmetrical three-line system is given by

$$[C]=\begin{bmatrix} c_{11} & -c_{12} & -c_{13} \\ -c_{12} & c_{22} & -c_{12} \\ -c_{13} & -c_{12} & c_{11} \end{bmatrix} \tag{A1}$$

where the C's are the per unit length self- and mutual-capacitance coefficients of the system.

For each of the three modes of propagation the capacitance (per unit length) for each of the three lines can be determined by using the method outlined in [2] in conjunction with (2) and the capacitance matrix given by (A1). These capacitances can be easily found to be

$$\text{for mode } A \quad C_A=c_{11}+c_{13}$$

$$\text{for mode } B \quad C_B=c_{11}-m_1c_{12}-c_{13}$$

$$\text{for mode } C \quad C_C=c_{11}+\frac{2}{m_1}c_{12}-c_{13}.$$

The above capacitances are the same for the three lines since their characteristic impedances have been set equal for each of the propagation modes.

If v_A, v_B, and v_C are the propagation velocities for modes A, B, and C, respectively, then the corresponding mode impedances for each of the three lines are given by

$$Z_A = \frac{1}{v_A(c_{11}+c_{13})} \tag{A2}$$

$$Z_B = \frac{1}{v_B(c_{11}-m_1c_{12}-c_{13})} \tag{A3}$$

$$Z_C = \frac{1}{v_C\left(c_{11}+\dfrac{2}{m_1}c_{12}-c_{13}\right)}. \tag{A4}$$

II. *Investigating the possibility of meeting the conditions* $\Gamma_A=0$ *and* $\Gamma_B=-\Gamma_C$ *for the system under consideration*:

The requirements of matching at all ports (i.e., $S_{11}=S_{22}=S_{33}=S_{44}=S_{55}=S_{66}=0$) and isolation between nonadjacent ports (i.e., $S_{13}=S_{31}=S_{46}=S_{64}=0$) can be met if $\Gamma_A=0$ and $\Gamma_B=-\Gamma_C$. For $\Gamma_A=0$, we need that $Z_A=Z_0$, and for $\Gamma_B=-\Gamma_C$, we need that $Z_BZ_C=Z_0^2$. But since $Z_A=Z_0$, then the required condition is

$$Z_BZ_C=Z_A^2. \tag{A5}$$

From (17), which was derived with $\Gamma_B=-\Gamma_C$, the value of m_1 becomes $\sqrt{2}$ for $\Gamma_A=0$ ($Z_A=Z_0$). When this value of m_1 is substituted in (A3) and (A4), we get for the left- and right-hand sides of (A5)

$$Z_BZ_C = \frac{1}{v_Bv_C\left[(c_{11}-c_{13})^2-2C_{12}^2\right]} \tag{A6}$$

and

$$Z_A^2 = \frac{1}{v_A^2(c_{11}+c_{13})^2}. \tag{A7}$$

For a quasi-homogeneous medium ($v_A \simeq v_B \simeq v_C$) the right-hand sides of (A6) and (A7) cannot be equal, and thus the abovementioned requirements cannot be met for the considered three-line system.

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